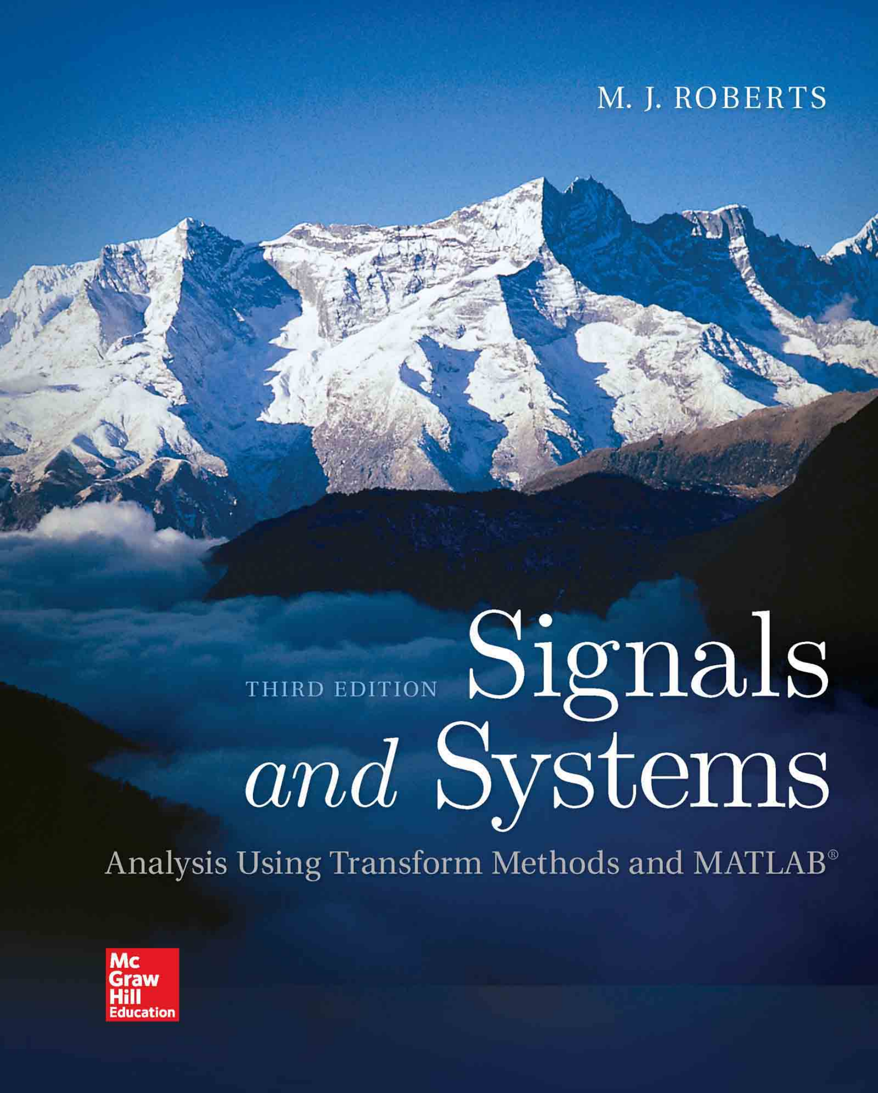

M. J. ROBERTS

THIRD EDITION

Signals *and* Systems

Analysis Using Transform Methods and MATLAB®

McGraw Hill Education

Signals and Systems

Analysis Using Transform Methods and MATLAB®

Third Edition

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SIGNALS AND SYSTEMS: ANALYSIS USING TRANSFORM METHODS AND MATLAB®, THIRD EDITION

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**To my wife Barbara for giving me the time and space to complete this effort
and to the memory of my parents, Bertie Ellen Pinkerton and Jesse Watts Roberts,
for their early emphasis on the importance of education.**

CONTENTS

PREFACE

MOTIVATION

I wrote the first and second editions because I love the mathematical beauty of signal and system analysis. That has not changed. The motivation for the third edition is to further refine the book structure in light of reviewers, comments, correct a few errors from the second edition and significantly rework the exercises.

AUDIENCE

This book is intended to cover a two-semester course sequence in the basics of signal and system analysis during the junior or senior year. It can also be used (as I have used it) as a book for a quick one-semester Master's-level review of transform methods as applied to linear systems.

CHANGES FROM THE SECOND EDITION

1. In response to reviewers, comments, two chapters from the second edition have been omitted: Communication Systems and State-Space Analysis. There seemed to be very little if any coverage of these topics in actual classes.
2. The second edition had 550 end-of-chapter exercises in 16 chapters. The third edition has 710 end-of-chapter exercises in 14 chapters.

OVERVIEW

Except for the omission of two chapters, the third edition structure is very similar to the second edition. The book begins with mathematical methods for describing signals and systems, in both continuous and discrete time. I introduce the idea of a transform with the continuous-time Fourier series, and from that base move to the Fourier transform as an extension of the Fourier series to aperiodic signals. Then I do the same for discrete-time signals. I introduce the Laplace transform both as a generalization of the continuous-time Fourier transform for unbounded signals and unstable systems and as a powerful tool in system analysis because of its very close association with the eigenvalues and eigenfunctions of continuous-time linear systems. I take a similar path for discrete-time systems using the z transform. Then I address sampling, the relation between continuous and discrete time. The rest of the book is devoted to applications in frequency-response analysis, feedback systems, analog and digital filters. Throughout the book I present examples and introduce MATLAB functions and operations to implement the methods presented. A chapter-by-chapter summary follows.

CHAPTER SUMMARIES

CHAPTER 1

Chapter 1 is an introduction to the general concepts involved in signal and system analysis without any mathematical rigor. It is intended to motivate the student by demonstrating the ubiquity of signals and systems in everyday life and the importance of understanding them.

CHAPTER 2

Chapter 2 is an exploration of methods of mathematically describing continuous-time signals of various kinds. It begins with familiar functions, sinusoids and exponentials and then extends the range of signal-describing functions to include continuous-time singularity functions (switching functions). Like most, if not all, signals and systems textbooks, I define the unit-step, the signum, the unit-impulse and the unit-ramp functions. In addition to these I define a unit rectangle and a unit periodic impulse function. The unit periodic impulse function, along with convolution, provides an especially compact way of mathematically describing arbitrary periodic signals.

After introducing the new continuous-time signal functions, I cover the common types of signal transformations, amplitude scaling, time shifting, time scaling, differentiation and integration and apply them to the signal functions. Then I cover some characteristics of signals that make them invariant to certain transformations, evenness, oddness and periodicity, and some of the implications of these signal characteristics in signal analysis. The last section is on signal energy and power.

CHAPTER 3

Chapter 3 follows a path similar to Chapter 2 except applied to discrete-time signals instead of continuous-time signals. I introduce the discrete-time sinusoid and exponential and comment on the problems of determining period of a discrete-time sinusoid. This is the first exposure of the student to some of the implications of sampling. I define some discrete-time signal functions analogous to continuous-time singularity functions. Then I explore amplitude scaling, time shifting, time scaling, differencing and accumulation for discrete-time signal functions pointing out the unique implications and problems that occur, especially when time scaling discrete-time functions. The chapter ends with definitions and discussion of signal energy and power for discrete-time signals.

CHAPTER 4

This chapter addresses the mathematical description of systems. First I cover the most common forms of classification of systems, homogeneity, additivity, linearity, time invariance, causality, memory, static nonlinearity and invertibility. By example I present various types of systems that have, or do not have, these properties and how to prove various properties from the mathematical description of the system.

CHAPTER 5

This chapter introduces the concepts of impulse response and convolution as components in the systematic analysis of the response of linear, time-invariant systems. I present the mathematical properties of continuous-time convolution and a graphical method of understanding what the convolution integral says. I also show how the properties of convolution can be used to combine subsystems that are connected in cascade or parallel into one system and what the impulse response of the overall system must be. Then I introduce the idea of a transfer function by finding the response of an LTI system to complex sinusoidal excitation. This section is followed by an analogous coverage of discrete-time impulse response and convolution.

CHAPTER 6

This is the beginning of the student's exposure to transform methods. I begin by graphically introducing the concept that any continuous-time periodic signal with engineering usefulness can be expressed by a linear combination of continuous-time sinusoids, real or complex. Then I formally derive the Fourier series using the concept of orthogonality to show where the signal description as a function of discrete harmonic number (the harmonic function) comes from. I mention the Dirichlet conditions to let the student know that the continuous-time Fourier series applies to all practical continuous-time signals, but not to all imaginable continuous-time signals.

Then I explore the properties of the Fourier series. I have tried to make the Fourier series notation and properties as similar as possible and analogous to the Fourier transform, which comes later. The harmonic function forms a "Fourier series pair" with the time function. In the first edition I used a notation for harmonic function in which lower-case letters were used for time-domain quantities and upper-case letters for their harmonic functions. This unfortunately caused some confusion because continuous- and discrete-time harmonic functions looked the same. In this edition I have changed the harmonic function notation for continuous-time signals to make it easily distinguishable. I also have a section on the convergence of the Fourier series illustrating the Gibb's phenomenon at function discontinuities. I encourage students to use tables and properties to find harmonic functions and this practice prepares them for a similar process in finding Fourier transforms and later Laplace and z transforms.

The next major section of Chapter 6 extends the Fourier series to the Fourier transform. I introduce the concept by examining what happens to a continuous-time Fourier series as the period of the signal approaches infinity and then define and derive the continuous-time Fourier transform as a generalization of the continuous-time Fourier series. Following that I cover all the important properties of the continuous-time Fourier transform. I have taken an "ecumenical" approach to two different notational conventions that are commonly seen in books on signals and systems, control systems, digital signal processing, communication systems and other applications of Fourier methods such as image processing and Fourier optics: the use of either cyclic frequency, f or radian frequency, ω. I use both and emphasize that the two are simply related through a change of variable. I think this better prepares students for seeing both forms in other books in their college and professional careers.

CHAPTER 7

This chapter introduces the discrete-time Fourier series (DTFS), the discrete Fourier transform (DFT) and the discrete-time Fourier transform (DTFT), deriving and defining them in a manner analogous to Chapter 6. The DTFS and the DFT are almost identical. I concentrate on the DFT because of its very wide use in digital signal processing. I emphasize the important differences caused by the differences between continuous- and discrete-time signals, especially the finite summation range of the DFT as opposed to the (generally) infinite summation range in the CTFS. I also point out the importance of the fact that the DFT relates a finite set of numbers to another finite set of numbers, making it amenable to direct numerical machine computation. I discuss the fast Fourier transform as a very efficient algorithm for computing the DFT. As in Chapter 6, I use both cyclic and radian frequency forms, emphasizing the relationships between them. I use F and Ω for discrete-time frequencies to distinguish them from f and ω, which were used in continuous time. Unfortunately, some authors reverse these symbols. My usage is more consistent with the majority of signals and systems texts. This is another example of the lack of standardization of notation in this area. The last major section is a comparison of the four Fourier methods. I emphasize particularly the duality between sampling in one domain and periodic repetition in the other domain.

CHAPTER 8

This chapter introduces the Laplace transform. I approach the Laplace transform from two points of view, as a generalization of the Fourier transform to a larger class of signals and as result which naturally follows from the excitation of a linear, time-invariant system by a complex exponential signal. I begin by defining the bilateral Laplace transform and discussing significance of the region of convergence. Then I define the unilateral Laplace transform. I derive all the important properties of the Laplace transform. I fully explore the method of partial-fraction expansion for finding inverse transforms and then show examples of solving differential equations with initial conditions using the unilateral form.

CHAPTER 9

This chapter introduces the z transform. The development parallels the development of the Laplace transform except applied to discrete-time signals and systems. I initially define a bilateral transform and discuss the region of convergence. Then I define a unilateral transform. I derive all the important properties and demonstrate the inverse transform using partial-fraction expansion and the solution of difference equations with initial conditions. I also show the relationship between the Laplace and z transforms, an important idea in the approximation of continuous-time systems by discrete-time systems in Chapter 14.

CHAPTER 10

This is the first exploration of the correspondence between a continuous-time signal and a discrete-time signal formed by sampling it. The first section covers how sampling is usually done in real systems using a sample-and-hold and an A/D converter. The second section starts by asking the question of how many samples are enough to describe a continuous-time signal. Then the question is answered by deriving the sampling theorem. Then I discuss interpolation methods, theoretical and practical, the special properties of bandlimited periodic signals. I do a complete development of the relationship between the CTFT of a continuous-time signal and DFT of a finite-length set of samples taken from it. Then I show how the DFT can be used to approximate the CTFT of an energy signal or a periodic signal. The next major section explores the use of the DFT in numerically approximating various common signal-processing operations.

CHAPTER 11

This chapter covers various aspects of the use of the CTFT and DTFT in frequency response analysis. The major topics are ideal filters, Bode diagrams, practical passive and active continuous-time filters and basic discrete-time filters.

CHAPTER 12

This chapter is on the application of the Laplace transform including block diagram representation of systems in the complex frequency domain, system stability, system interconnections, feedback systems including root locus, system responses to standard signals and lastly standard realizations of continuous-time systems.

CHAPTER 13

This chapter is on the application of the z transform including block diagram representation of systems in the complex frequency domain, system stability, system interconnections, feedback systems including root-locus, system responses to standard signals, sampled-data systems and standard realizations of discrete-time systems.

CHAPTER 14

This chapter covers the analysis and design of some of the most common types of practical analog and digital filters. The analog filter types are Butterworth, Chebyshev Types 1 and 2 and Elliptic (Cauer) filters. The section on digital filters covers the most common types of techniques for simulation of analog filters including, impulse- and step-invariant, finite difference, matched z transform, direct substitution, bilinear z transform, truncated impulse response and Parks-McClellan numerical design.

APPENDICES

There are seven appendices on useful mathematical formulae, tables of the four Fourier transforms, Laplace transform tables and z transform tables.

CONTINUITY

The book is structured so as to facilitate skipping some topics without loss of continuity. Continuous-time and discrete-time topics are covered alternately and continuous-time analysis could be covered without reference to discrete time. Also, any or all of the last six chapters could be omitted in a shorter course.

REVIEWS AND EDITING

This book owes a lot to the reviewers, especially those who really took time and criticized and suggested improvements. I am indebted to them. I am also indebted to the many students who have endured my classes over the years. I believe that our relationship is more symbiotic than they realize. That is, they learn signal and system analysis from me and I learn how to teach signal and system analysis from them. I cannot count the number of times I have been asked a very perceptive question by a student that revealed not only that the students were not understanding a concept but that I did not understand it as well as I had previously thought.

WRITING STYLE

Every author thinks he has found a better way to present material so that students can grasp it and I am no different. I have taught this material for many years and through the experience of grading tests have found what students generally do and do not grasp. I have spent countless hours in my office one-on-one with students explaining these concepts to them and, through that experience, I have found out what needs to be said. In my writing I have tried to simply speak directly to the reader in a straightforward conversational way, trying to avoid off-putting formality and, to the extent possible, anticipating the usual misconceptions and revealing the fallacies in them. Transform methods are not an obvious idea and, at first exposure, students can easily get bogged down in a bewildering morass of abstractions and lose sight of the goal, which is to analyze a system's response to signals. I have tried (as every author does) to find the magic combination of accessibility and mathematical rigor because both are important. I think my writing is clear and direct but you, the reader, will be the final judge of whether or not that is true.

EXERCISES

Each chapter has a group of exercises along with answers and a second group of exercises without answers. The first group is intended more or less as a set of "drill" exercises and the second group as a set of more challenging exercises.

CONCLUDING REMARKS

As I indicated in the preface to first and second editions, I welcome any and all criticism, corrections and suggestions. All comments, including ones I disagree with and ones which disagree with others, will have a constructive impact on the next edition because they point out a problem. If something does not seem right to you, it probably will bother others also and it is my task, as an author, to find a way to solve that problem. So I encourage you to be direct and clear in any remarks about what you believe should be changed and not to hesitate to mention any errors you may find, from the most trivial to the most significant.

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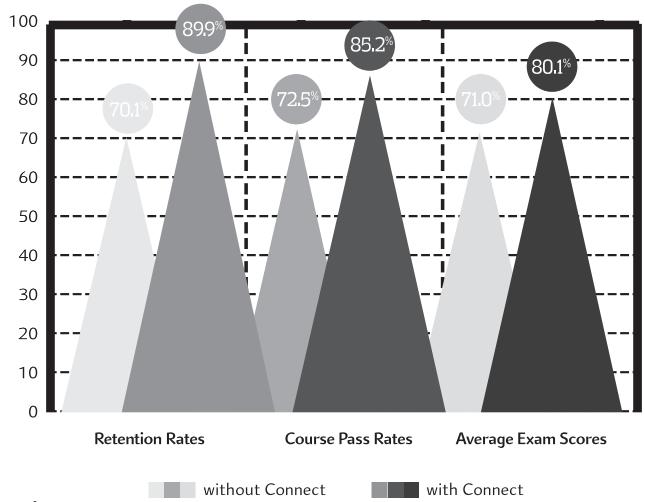

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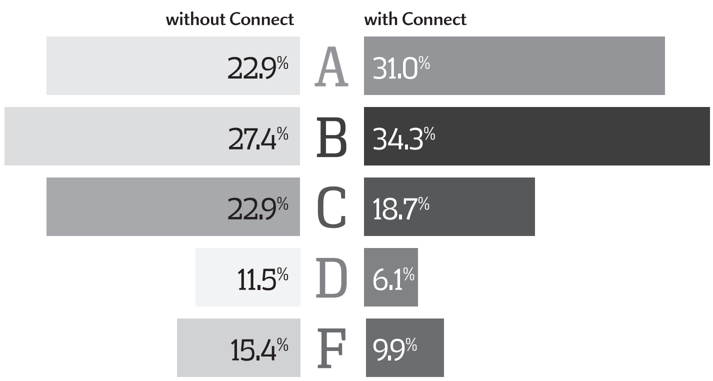

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CHAPTER 1

Introduction

1.1 SIGNALS AND SYSTEMS DEFINED

Any time-varying physical phenomenon that is intended to convey information is a **signal**. Examples of signals are the human voice, sign language, Morse code, traffic signals, voltages on telephone wires, electric fields emanating from radio or television transmitters, and variations of light intensity in an optical fiber on a telephone or computer network. **Noise** is like a signal in that it is a time-varying physical phenomenon, but usually it does not carry useful information and is considered undesirable.

Signals are operated on by **systems**. When one or more **excitations** or **input signals** are applied at one or more system **inputs**, the system produces one or more **responses** or **output signals** at its **outputs**. Figure 1.1 is a block diagram of a single-input, single-output system.

Figure 1.1
Block diagram of a single-input, single-output system

In a communication system, a transmitter produces a signal and a receiver acquires it. A **channel** is the path a signal takes from a transmitter to a receiver. Noise is inevitably introduced into the transmitter, channel and receiver, often at multiple points (Figure 1.2). The transmitter, channel and receiver are all components or subsystems of the overall system. Scientific instruments are systems that measure a physical phenomenon (temperature, pressure, speed, etc.) and convert it to a voltage or current, a signal. Commercial building control systems (Figure 1.3), industrial plant control systems (Figure 1.4), modern farm machinery (Figure 1.5), avionics in airplanes, ignition and fuel pumping controls in automobiles, and so on are all systems that operate on signals.

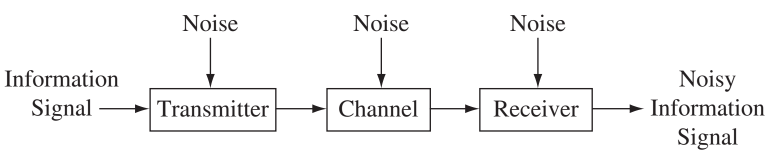

Figure 1.2
A communication system

Figure 1.3
Modern office buildings

Figure 1.4
Typical industrial plant control room

Figure 1.5
Modern farm tractor with enclosed cab

The term *system* even encompasses things such as the stock market, government, weather, the human body and the like. They all respond when excited. Some systems are readily analyzed in detail, some can be analyzed approximately, but some are so complicated or difficult to measure that we hardly know enough to understand them.

1.2 TYPES OF SIGNALS

There are several broad classifications of signals: **continuous-time**, **discrete-time**, **continuous-value**, **discrete-value**, **random** and **nonrandom**. A continuous-time signal is defined at every instant of time over some time interval. Another common name for some continuous-time signals is **analog** signal, in which the variation of the signal with time is *analogous* (proportional) to some physical phenomenon. All analog signals are continuous-time signals but not all continuous-time signals are analog signals (Figure 1.6 through Figure 1.8).

Sampling a signal is acquiring values from a continuous-time signal at discrete points in time. The set of samples forms a discrete-time signal. A discrete-time signal

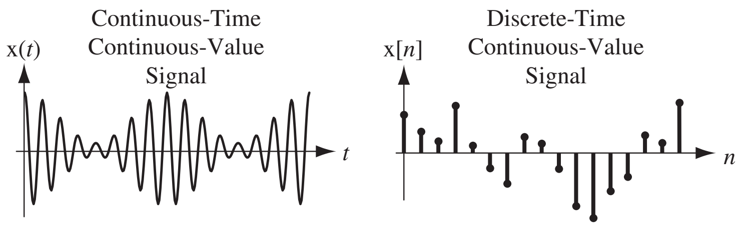

Figure 1.6
Examples of continuous-time and discrete-time signals

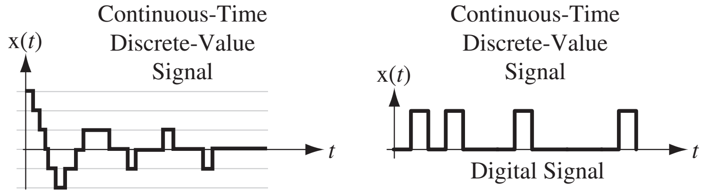

Figure 1.7
Examples of continuous-time, discrete-value signals

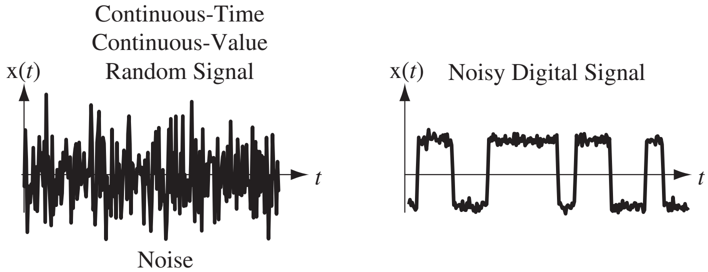

Figure 1.8
Examples of noise and a noisy digital signal

can also be created by an inherently discrete-time system that produces signal values only at discrete times (Figure 1.6).

A continuous-value signal is one that may have any value within a continuum of allowed values. In a continuum any two values can be arbitrarily close together. The real numbers form a continuum with infinite extent. The real numbers between zero and one form a continuum with finite extent. Each is a set with infinitely many members (Figure 1.6 through Figure 1.8).

A discrete-value signal can only have values taken from a discrete set. In a discrete set of values the magnitude of the difference between any two values is greater than some positive number. The set of integers is an example. Discrete-time signals are usually transmitted as **digital** signals, a sequence of values of a discrete-time signal in the form of digits in some encoded form. The term *digital* is also sometimes used loosely to refer to a discrete-value signal that has only two possible values. The digits in this type of digital signal are transmitted by signals that are continuous-time. In this case, the terms *continuous-time* and *analog* are not synonymous. A digital signal of this type is a continuous-time signal but not an analog signal because its variation of value with time is not directly analogous to a physical phenomenon (Figure 1.6 through Figure 1.8).

A random signal cannot be predicted exactly and cannot be described by any mathematical function. A **deterministic** signal can be mathematically described. A common name for a random signal is **noise** (Figure 1.6 through Figure 1.8).

In practical signal processing it is very common to acquire a signal for processing by a computer by sampling, **quantizing** and **encoding** it (Figure 1.9). The original signal is a continuous-value, continuous-time signal. Sampling acquires its values at discrete times and those values constitute a continuous-value, discrete-time signal. Quantization approximates each sample as the nearest member of a finite set of discrete values, producing a discrete-value, discrete-time signal. Each signal value in the set of discrete values at discrete times is converted to a sequence of rectangular pulses that encode it into a binary number, creating a discrete-value, continuous-time signal, commonly called a *digital signal*. The steps illustrated in Figure 1.9 are usually carried out by a single device called an **analog-to-digital converter (ADC)**.

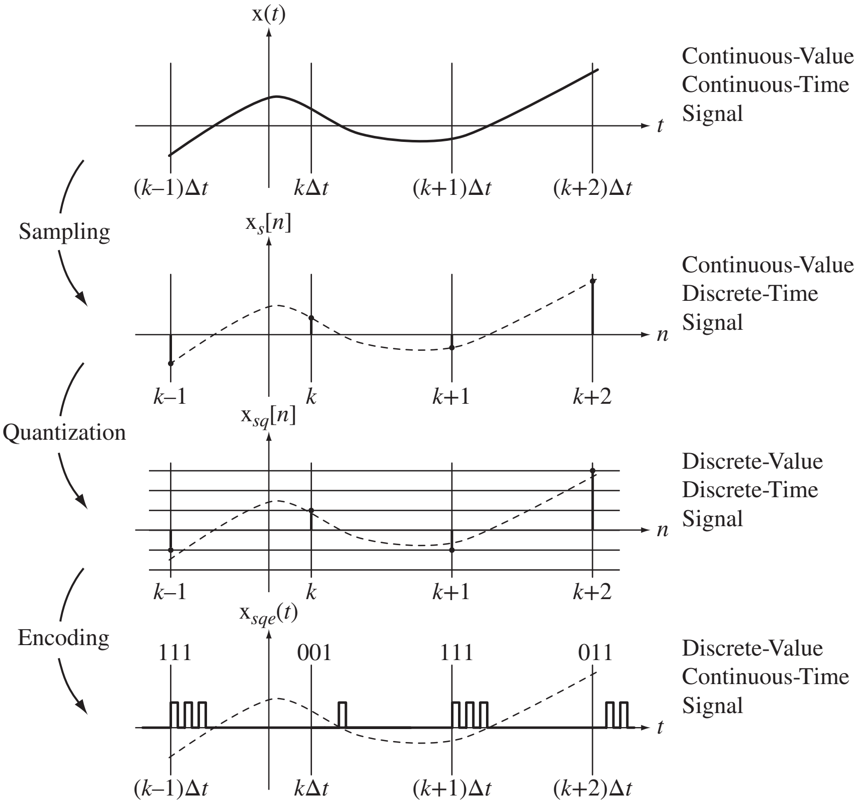

Figure 1.9
Sampling, quantization and encoding of a signal to illustrate various signal types

One common use of binary digital signals is to send text messages using the American Standard Code for Information Interchange (ASCII). The letters of the alphabet, the digits 0–9, some punctuation characters and several nonprinting control characters, for a total of 128 characters, are all encoded into a sequence of 7 binary bits. The 7 bits are sent sequentially, preceded by a **start** bit and followed by 1 or 2 **stop** bits for synchronization purposes. Typically, in direct-wired connections between digital equipment, the bits are represented by a higher voltage (2 to 5 V) for a 1 and a lower voltage level (around 0 V) for a 0. In an asynchronous transmission using one start and one stop bit, sending the message SIGNAL, the voltage versus time would look as illustrated in Figure 1.10.

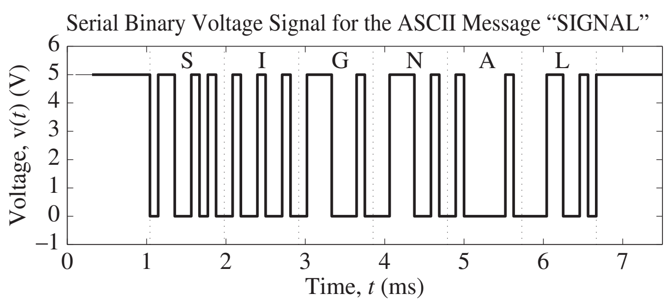

Figure 1.10
Asynchronous serial binary ASCII-encoded voltage signal for the word SIGNAL

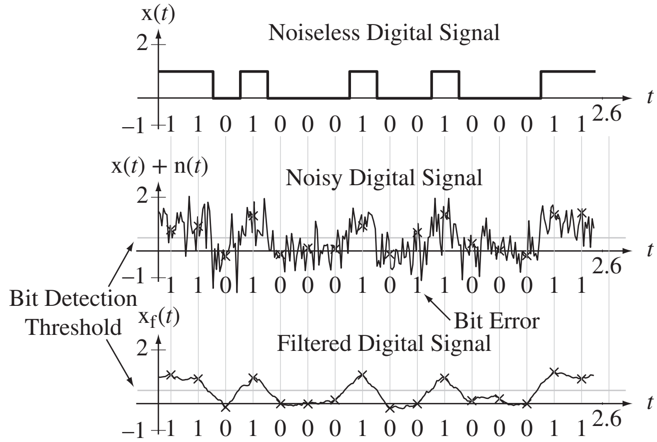

Figure 1.11
Use of a filter to reduce bit error rate in a digital signal

In 1987 ASCII was extended to Unicode. In Unicode the number of bits used to represent a character can be 8, 16, 24 or 32 and more than 120,000 characters are currently encoded in modern and historic language characters and multiple symbol sets.

Digital signals are important in signal analysis because of the spread of digital systems. Digital signals often have better immunity to noise than analog signals. In binary signal communication the bits can be detected very cleanly until the noise gets very large. The detection of bit values in a stream of bits is usually done by comparing the signal value at a predetermined bit time with a threshold. If it is above the threshold it is declared a 1 and if it is below the threshold it is declared a 0. In Figure 1.11, the x's mark the signal value at the detection time, and when this technique is applied to the noisy digital signal, one of the bits is incorrectly detected. But when the signal is processed by a **filter**, all the bits are correctly detected. The filtered digital signal does not look very clean in comparison with the noiseless digital signal, but the bits can still be detected with a very low probability of error. This is the basic reason that digital signals can have better noise immunity than analog signals. An introduction to the analysis and design of filters is presented in Chapters 11 and 15.

In this text we will consider both continuous-time and discrete-time signals, but we will (mostly) ignore the effects of signal quantization and consider all signals to be continuous-value. Also, we will not directly consider the analysis of random signals, although random signals will sometimes be used in illustrations.

The first signals we will study are continuous-time signals. Some continuous-time signals can be described by continuous functions of time. A signal $x(t)$ might be described by a function $x(t) = 50\sin(200\pi t)$ of continuous time t. This is an exact description of the signal at every instant of time. The signal can also be described graphically (Figure 1.12).

Many continuous-time signals are not as easy to describe mathematically. Consider the signal in Figure 1.13. Waveforms like the one in Figure 1.13 occur in various types of instrumentation and communication systems. With the definition of some signal functions and an operation called **convolution,** this signal can be compactly described, analyzed and manipulated mathematically. Continuous-time signals that can be described by mathematical functions can be transformed into another domain called the **frequency domain** through the **continuous-time Fourier transform**. In this context, **transformation** means transformation of a signal to the frequency domain. This is an important tool in signal analysis, which allows certain characteristics of the signal to be more clearly observed

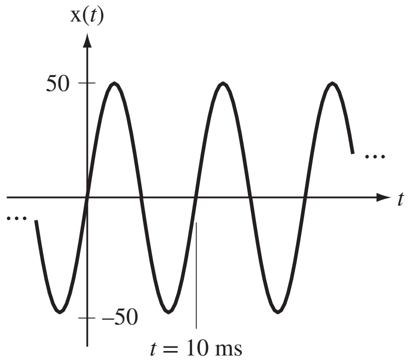

Figure 1.12
A continuous-time signal described by a mathematical function

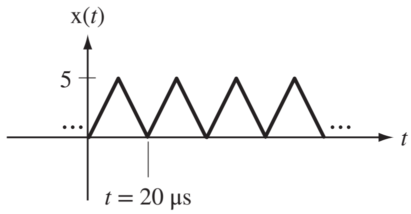

Figure 1.13
A second continuous-time signal

and more easily manipulated than in the time domain. (In the frequency domain, signals are described in terms of the frequencies they contain.) Without frequency-domain analysis, design and analysis of many systems would be considerably more difficult.

Discrete-time signals are only defined at discrete points in time. Figure 1.14 illustrates some discrete-time signals.

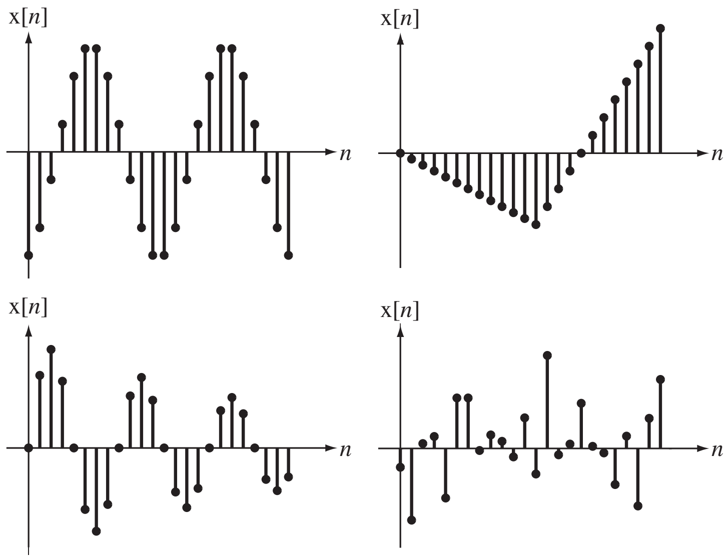

Figure 1.14
Some discrete-time signals

So far all the signals we have considered have been described by functions of time. An important class of "signals" is functions of **space** instead of time: images. Most of the theories of signals, the information they convey and how they are processed by systems in this text will be based on signals that are a variation of a physical phenomenon with time. But the theories and methods so developed also apply, with only minor modifications, to the processing of images. Time signals are described by the variation of a physical phenomenon as a function of a single independent variable, time. Spatial signals, or images, are described by the variation of a physical phenomenon as a

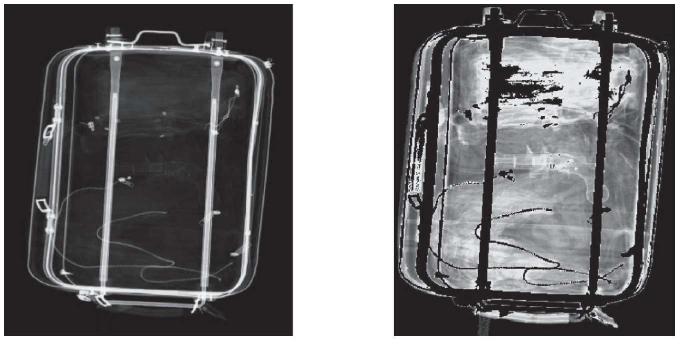

Figure 1.15
An example of image processing to reveal information
(Original X-ray image and processed version provided by the Imaging, Robotics and Intelligent Systems (IRIS) Laboratory of the Department of Electrical and Computer Engineering at the University of Tennessee, Knoxville.)

function of two orthogonal, independent, **spatial** variables, conventionally referred to as x and y. The physical phenomenon is most commonly light or something that affects the transmission or reflection of light, but the techniques of image processing are also applicable to anything that can be mathematically described by a function of two independent variables.

Historically the practical application of image-processing techniques has lagged behind the application of signal-processing techniques because the amount of information that has to be processed to gather the information from an image is typically much larger than the amount of information required to get the information from a time signal. But now image processing is increasingly a practical technique in many situations. Most image processing is done by computers. Some simple image-processing operations can be done directly with optics and those can, of course, be done at very high speeds (at the speed of light!). But direct optical image processing is very limited in its flexibility compared with digital image processing on computers.

Figure 1.15 shows two images. On the left is an unprocessed X-ray image of a carry-on bag at an airport checkpoint. On the right is the same image after being processed by some image-filtering operations to reveal the presence of a weapon. This text will not go into image processing in any depth but will use some examples of image processing to illustrate concepts in signal processing.

An understanding of how signals carry information and how systems process signals is fundamental to multiple areas of engineering. Techniques for the analysis of signals processed by systems are the subject of this text. This material can be considered as an applied mathematics text more than a text covering the building of useful devices, but an understanding of this material is very important for the successful design of useful devices. The material that follows builds from some fundamental definitions and concepts to a full range of analysis techniques for continuous-time and discrete-time signals in systems.

1.3 EXAMPLES OF SYSTEMS

There are many different types of signals and systems. A few examples of systems are discussed next. The discussion is limited to the qualitative aspects of the system with some illustrations of the behavior of the system under certain conditions. These systems will be revisited in Chapter 4 and discussed in a more detailed and quantitative way in the material on system modeling.

A MECHANICAL SYSTEM

A man bungee jumps off a bridge over a river. Will he get wet? The answer depends on several factors:

1. The man's height and weight
2. The height of the bridge above the water
3. The length and springiness of the bungee cord

When the man jumps off the bridge he goes into free fall caused by the force due to gravitational attraction until the bungee cord extends to its full unstretched length. Then the system dynamics change because there is now another force on the man, the bungee cord's resistance to stretching, and he is no longer in free fall. We can write and solve a differential equation of motion and determine how far down the man falls before the bungee cord pulls him back up. The differential equation of motion is a **mathematical model** of this mechanical system. If the man weighs 80 kg and is 1.8 m tall, and if the bridge is 200 m above the water level and the bungee cord is 30 m long (unstretched) with a spring constant of 11 N/m, the bungee cord is fully extended before stretching at $t = 2.47$ s. The equation of motion, after the cord starts stretching, is

$$\mathrm{x}(t) = -16.85 \sin(0.3708t) - 95.25 \cos(0.3708t) + 101.3, \quad t > 2.47. \quad \textbf{(1.1)}$$

Figure 1.16 shows his position versus time for the first 15 seconds. From the graph it seems that the man just missed getting wet.

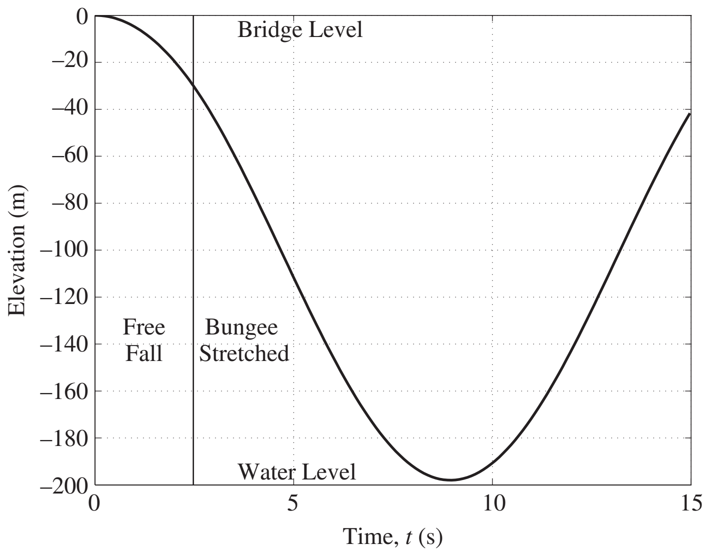

Figure 1.16
Man's vertical position versus time (bridge level is zero)

A FLUID SYSTEM

A fluid system can also be modeled by a differential equation. Consider a cylindrical water tank being fed by an input flow of water, with an orifice at the bottom through which flows the output (Figure 1.17).

The flow out of the orifice depends on the height of the water in the tank. The variation of the height of the water depends on the input flow and the output flow. The rate